



MIDLAND PARK PUBLIC SCHOOLS
Midland Park, New Jersey
CURRICULUM

Precalculus

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Approved by the Midland Park Board of Education on
July 11, 2017

Grade 11/12 Pre-Calculus Curriculum Overview

Grade 11/12 Pre-Calculus is taught in seven units throughout the school year. The Pre-Calculus curriculum is a rigorous, open-ended and sequential process of connecting the trigonometric and calculus material to previously learned algebraic topics. As part of the spiraling curriculum, aspects of Algebra, Geometry, and Algebra II are taught throughout the year. A guided inquiry program gives students the opportunity to explore topics and concepts through mathematical investigations. Participating in this rigorous program helps students:

1. To foster a lifelong enjoyment of learning mathematics.
2. To observe mathematics in the world around them.
3. To meet the mathematics standards for New Jersey Public Schools.

Suggested Course Sequence*:

Unit 1: Trigonometric Functions and Their Graphs: 27 days

Unit 2: Trigonometric Identities: 17 days

Unit 3: Oblique Triangles and Vectors: 18 days

Unit 4: Linear Systems and Matrices: 25 days

Unit 5: Sequences, Series, and Probability: 20 days

Unit 6: Conic Sections: 29 days

Unit 7: Introduction to Limits: 19 days

Pre-Requisite: Algebra II

**The number of instructional days is an estimate based on the information available at this time. 1 day equals approximately 48 minutes of seat time. Teachers are strongly encouraged to review the entire unit of study carefully and collaboratively to determine whether adjustments to this estimate need to be made.*

Content Area: Pre-Calculus	
Unit Title: Unit #1- Trigonometric Functions and Their Graphs	
Grade Level: 11th-12th	
<p>Unit Summary: In this unit, students will be using right triangle trigonometry to create the unit circle and use the trigonometric functions to find and evaluate angle measures. Students will be evaluating and graphing the six trigonometric functions, using the unit circle. They will be comparing these functions to the corresponding inverse functions, as well as their reciprocal functions.</p> <p>Interdisciplinary Connections: Sports, Architecture, Various topics in Science, Finance and Accounting</p> <p>21st Century Themes and Skills: Creativity and Innovation, Communication and Collaboration, Critical Thinking & Problem Solving, Information, Media, and Technology Skills, Life and Career Skills.</p> <p>CRP1. Act as a responsible and contributing citizen and employee. CRP2. Apply appropriate academic and technical skills. CRP4. Communicate clearly and effectively. CRP6. Demonstrate creativity and innovation. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP9. Model integrity, ethical leadership and effective management. CRP11. Use technology to enhance productivity.</p>	
Standards (Content and Technology):	
CPI#:	Statement:
NJSLS.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
NJSLS.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
NJSLS.F-TF.A.3(+)	(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x , $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.
NJSLS.F-TF.A.4(+)	(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
NJSLS.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
NJSLS.F-TF.B.6(+)	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
NJSLS.F-TF.B.7(+)	(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
Tech Standard: 8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
Tech Standard: 8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.
Tech Standard: 8.2.12.C.4	Explain and identify interdependent systems and their functions.
Unit Essential Question(s): <ul style="list-style-type: none"> ● How do you describe angles and angular movement? ● How do you evaluate trigonometric functions by using the unit circle? 	Unit Enduring Understandings: <ul style="list-style-type: none"> ● Reciprocal and inverse are different processes ● Trigonometric identities are true in both radian and degree modes

<ul style="list-style-type: none"> • How do you use trigonometry to find unknown side lengths and angle measures in right triangles? • How do you evaluate trigonometric functions of any angle? • How do you sketch the graphs of sine, cosine, and other trigonometric functions? • How do you evaluate and graph the inverses of trigonometric functions? • How do you use trigonometric functions to solve real-life problems? 	<ul style="list-style-type: none"> • The reference angle is the acute angle formed with the horizontal • Use reference angles when evaluating trigonometric functions of angles greater than 90° • To obtain the correct graphs of the trigonometric functions, calculators should be set to radian mode • The inverse sine function $y = \arcsin x$ or $y = \sin^{-1}x$, can be stated as the phrase “the angle (or number) whose sine is x”. • The values of the inverse sine function are always in radians • The range of each inverse trigonometric function is limited to allow it to be a function, and each range is different.
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Unit Learning Targets/Objectives:
Students will...

- Be able to use radian measures and the definitions of trigonometric functions on the unit circle.
- Be able to find ratios of an acute angle by drawing a triangle, and of any angle by drawing a unit circle and a reference angle.
- Be able to graph trigonometric functions, their reciprocals, and their inverses and identify the basic characteristics of the trigonometric functions
- Be able to use trigonometric ratios to solve problems in a variety of contexts, such as mechanics, biology, and navigation.

Formative Assessments:

- Quizzes
- Homework
- On spot checking for understanding activities
- Entry/Exit tickets

Summative/Benchmark Assessment(s):

- Tests
- Projects

Resources/Materials (copy hyperlinks for digital resources):
Larson Precalculus with Limits Teacher Resources
www.njctl.org/courses/math/pre-calculus/

Modifications:

<ul style="list-style-type: none"> • Special Education Students <ul style="list-style-type: none"> ○ Allow errors ○ Rephrase questions, directions, and explanations ○ Allow extended time to answer questions, and permit drawing, as an explanation ○ Accept participation at any level, even one word ○ Consult with Case Managers and follow IEP accommodations/modifications • English Language Learners 	<ul style="list-style-type: none"> • At-Risk Students <ul style="list-style-type: none"> ○ Provide extended time to complete tasks ○ Consult with Guidance Counselors and follow I&RS procedures/action plans ○ Consult with classroom teacher(s) for specific behavior interventions ○ Provide rewards as necessary • Gifted and Talented Students <ul style="list-style-type: none"> ○ Provide extension activities ○ Build on students’ intrinsic motivations
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- o Assign a buddy, same language or English speaking
 - o Allow errors in speaking
 - o Rephrase questions, directions, and explanations
 - o Allow extended time to answer questions
 - o Accept participation at any level, even one word
- o Consult with parents to accommodate students' interests in completing tasks at their level of engagement

Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)
Radian and Degree Measure	Students will be able to distinguish between radian and degree measures, and convert angles to both modes.	3 days
Trigonometric Functions: The Unit Circle	Students will be able to use radian measures and the definitions of trigonometric functions on the unit circle.	4 days
Right Triangle Trigonometry	Students will be able to find ratios of an acute angle by drawing a triangle, and of any angle by drawing a unit circle and a reference angle.	3 days
Trigonometric Functions of any Angle	Students will be able to evaluate trigonometric functions of any angle, focusing on angles that are not the foundation angles on the unit circle.	4 days
Graphs of Sine and Cosine Functions	Students will be able to graph the sine and cosine functions and identify the basic characteristics of the trigonometric functions.	4 days
Graphs of Other Trigonometric Functions	Students will be able to graph the other trigonometric functions and identify the basic characteristics of the trigonometric functions.	4 days
Inverse Trigonometric Functions	Students will be able to evaluate and graph the inverse trigonometric functions.	3 days
Applications and Models	Students will be able to use trigonometric ratios to solve problems in a variety of contexts, such as mechanics, biology, and navigation.	2 days
<p>Teacher Notes:</p> <p>Additional Resources Click links below to access additional resources used to design this unit:</p>		

Content Area: Pre-Calculus	
Unit Title: Unit #2- Trigonometric Identities	
Grade Level: 11th-12th	
<p>Unit Summary: In this unit, students will learn different strategies for simplifying expressions and solving equations involving the trigonometric identities. Students will verify trigonometric identities using combinations of other identities along with properties of algebra. Students will also solve equations using the various sum and difference, multiple-angle, and product-to-sum formulas. Students will learn to recognize when each of these formulas is needed in order to solve the equation.</p> <p>Interdisciplinary Connections: Sports, Architecture, Various topics in Science, Finance and Accounting</p> <p>21st Century Themes and Skills: Creativity and Innovation, Communication and Collaboration, Critical Thinking & Problem Solving, Information, Media, and Technology Skills, Life and Career Skills.</p> <p>CRP1. Act as a responsible and contributing citizen and employee. CRP2. Apply appropriate academic and technical skills. CRP4. Communicate clearly and effectively. CRP6. Demonstrate creativity and innovation. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP9. Model integrity, ethical leadership and effective management. CRP11. Use technology to enhance productivity.</p>	
Standards (Content and Technology):	
CPI#:	Statement:
NJSLS.F-TF.C.9	(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems
Tech Standard: 8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
Tech Standard: 8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.
Tech Standard: 8.2.12.C.4	Explain and identify interdependent systems and their functions.
<p>Unit Essential Question(s):</p> <ul style="list-style-type: none"> ● How do you evaluate trigonometric expressions in order to simplify and evaluate trigonometric functions? ● How do you verify a trigonometric identity? ● How do you solve trigonometric equations written in quadratic form or containing more than one angle? ● How do you simplify expressions and solve equations that contain sums or differences of angles? ● How do you rewrite trigonometric expressions that contain functions of multiple or half-angles, or functions that involve squares or products of trigonometric expressions? 	<p>Unit Enduring Understandings:</p> <ul style="list-style-type: none"> ● Identities are two equivalent expressions. When verifying identities, either side must be worked on separately as an expression. ● When solving trigonometric equations, if no interval is stated, solutions must include multiples beyond the first period. ● When solving trigonometric equations, it is incorrect to divide by a trigonometric function since it may be zero.
<p>Unit Learning Targets/Objectives: <i>Students will...</i></p> <ul style="list-style-type: none"> ● Be able to rewrite trigonometric functions by using identities and will be able to verify identities by rewriting the functions. ● Be able to solve trigonometric equations written in quadratic form and equations containing more than one angle. 	

- Be able to solve equations containing sums and differences of angles.
- Be able to rewrite trigonometric functions of multiple angles, half-angles, or squares of products of trigonometric functions.

Formative Assessments:

- Quizzes
- Homework
- On spot checking for understanding activities
- Entry/Exit tickets

Summative/Benchmark Assessment(s):

- Tests
- Projects

Resources/Materials (copy hyperlinks for digital resources):

Larson Precalculus with Limits Teacher Resources
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Modifications:

- Special Education Students
 - Allow errors
 - Rephrase questions, directions, and explanations
 - Allow extended time to answer questions, and permit drawing, as an explanation
 - Accept participation at any level, even one word
 - Consult with Case Managers and follow IEP accommodations/modifications
- English Language Learners
 - Assign a buddy, same language or English speaking
 - Allow errors in speaking
 - Rephrase questions, directions, and explanations
 - Allow extended time to answer questions
 - Accept participation at any level, even one word
- At-Risk Students
 - Provide extended time to complete tasks
 - Consult with Guidance Counselors and follow I&RS procedures/action plans
 - Consult with classroom teacher(s) for specific behavior interventions
 - Provide rewards as necessary
- Gifted and Talented Students
 - Provide extension activities
 - Build on students' intrinsic motivations
 - Consult with parents to accommodate students' interests in completing tasks at their level of engagement

Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)
Using Fundamental Identities	Students will be able to rewrite trigonometric functions by using identities.	1 day
Verifying Trigonometric Identities	Students will be able to verify identities by rewriting the functions and simplifying the expressions.	4 days
Solving Trigonometric Identities	Students will be able to solve trigonometric equations written in quadratic form and equations	5 days

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	containing more than one angle, using the trigonometric identities.	
Sum and Difference Formula	Students will be able to solve equations containing sums and differences of angles.	3 days
Multiple-Angle and Product- to-Sum Formulas	Students will be able to rewrite trigonometric functions of multiple angles, half-angles, or squares of products of trigonometric functions.	4 days
<p>Teacher Notes:</p> <p>Additional Resources Click links below to access additional resources used to design this unit:</p>		

Content Area: Pre-Calculus	
Unit Title: Unit #3- Oblique Triangles and Vectors	
Grade Level: 11th-12th	
<p>Unit Summary: In this unit, students will study oblique triangles and learn the methods of solving for side lengths and angle measures of these triangles. They will discover the different criteria that is needed to use the Law of Sines and the Law of Cosines, and will also determine whether a given triangle has one solution, no solution, or two solutions. Students will also define vectors and use the component form of a vector to find its magnitude, perform vector operations, and find unit vectors. They will learn the significance of the dot product, and will also determine the angle measure between two vectors.</p>	
<p>Interdisciplinary Connections: Sports, Architecture, Various topics in Science, Finance and Accounting</p>	
<p>21st Century Themes and Skills: Creativity and Innovation, Communication and Collaboration, Critical Thinking & Problem Solving, Information, Media, and Technology Skills, Life and Career Skills.</p> <p>CRP1. Act as a responsible and contributing citizen and employee. CRP2. Apply appropriate academic and technical skills. CRP4. Communicate clearly and effectively. CRP6. Demonstrate creativity and innovation. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP9. Model integrity, ethical leadership and effective management. CRP11. Use technology to enhance productivity.</p>	
Standards (Content and Technology):	
CPI#:	Statement:
NJSLS.G-SRT.D.10(+)	(+) Prove the Laws of Sines and Cosines and use them to solve problems.
NJSLS.G-SRT.D.11(+)	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
NJSLS.N-VM.A.1(+)	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., \mathbf{v} , $ \mathbf{v} $)
NJSLS.N-VM.A.2(+)	(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
NJSLS.N-VM.A.3(+)	(+) Solve problems involving velocity and other quantities that can be represented by vectors.
NJSLS.N-VM.B.4(+).a.b.c	(+) Add and subtract vectors. A. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. B. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. C. Understand vector subtraction $\mathbf{v}-\mathbf{w}$ as $\mathbf{v}+(-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of \mathbf{w} , with the same magnitude as \mathbf{w} and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
NJSLS.N-VM.B.5(+).a.b	(+) Multiply a vector by a scalar. A. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. B. Compute the magnitude of a scalar multiple $c\mathbf{v}$ knowing that when $ c \neq 0$, the direction of $c\mathbf{v}$ is either along \mathbf{v} (for $c > 0$) or against \mathbf{v} (for $c < 0$).
Tech Standard: 8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.

Tech Standard: 8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.	
Tech Standard: 8.2.12.C.4	Explain and identify interdependent systems and their functions.	
Unit Essential Question(s): <ul style="list-style-type: none"> ● How do you use trigonometry to solve and find the areas of oblique triangles? ● How do you represent and perform operations with vector quantities? ● How do you write a vector as the sum of two vector components? ● How do you represent and perform operations on complex numbers by using trigonometry? 	Unit Enduring Understandings: <ul style="list-style-type: none"> ● The longest side of a triangle lies opposite the largest angle, and the height is perpendicular to its base. ● Use the Law of Sines to solve triangles in cases AAS, ASA, and SSA. ● The Ambiguous Case (SSA pattern) can have no solution, one solution, or two solutions. ● Use the Law of Cosines to solve and find unknown side lengths or angles in SSS and SAS cases. ● The component form of a vector represents a family of vectors. ● The \tan^{-1} function key will give an angle restricted to quadrant IV or I. The argument of the answer may need to be adjusted to reflect the correct quadrant. 	
Unit Learning Targets/Objectives: <i>Students will...</i> <ul style="list-style-type: none"> ● Be able to find side lengths, angles, and areas of oblique triangles by using the Law of Sines and the Law of Cosines. ● Be able to use vectors and trigonometry to solve real-world problems such as finding force on an inclined ramp or a wind adjusted bearing in airplane navigation. ● Be able to write and perform operations on complex numbers in trigonometric form. 		
Formative Assessments: <ul style="list-style-type: none"> ● Quizzes ● Homework ● On spot checking for understanding activities ● Entry/Exit tickets Summative/Benchmark Assessment(s): <ul style="list-style-type: none"> ● Tests ● Projects Resources/Materials (copy hyperlinks for digital resources): <i>Larson Precalculus with Limits</i> Teacher Resources www.njctl.org/courses/math/pre-calculus/		
Modifications: <ul style="list-style-type: none"> ● Special Education Students <ul style="list-style-type: none"> ○ Allow errors ○ Rephrase questions, directions, and explanations ○ Allow extended time to answer questions, and permit drawing, as an explanation ○ Accept participation at any level, even one word ○ Consult with Case Managers and follow IEP accommodations/modifications ● At-Risk Students <ul style="list-style-type: none"> ○ Provide extended time to complete tasks ○ Consult with Guidance Counselors and follow I&RS procedures/action plans ○ Consult with classroom teacher(s) for specific behavior interventions ○ Provide rewards as necessary ● Gifted and Talented Students <ul style="list-style-type: none"> ○ Provide extension activities 		

- English Language Learners
 - Assign a buddy, same language or English speaking
 - Allow errors in speaking
 - Rephrase questions, directions, and explanations
 - Allow extended time to answer questions
 - Accept participation at any level, even one word
- Build on students' intrinsic motivations
- Consult with parents to accommodate students' interests in completing tasks at their level of engagement

Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)
Law of Sines	Students will be able to use the Law of Sines to find side lengths, angles, and areas of oblique triangles.	4 days
Law of Cosines	Students will be able to use the Law of Cosines to find side lengths, angles, and areas of oblique triangles.	4 days
Vectors in the Plane	Students will be able to find component forms of vectors, find the magnitude of vectors, and will perform vector operations in the plane.	5 days
Vectors and Dot Products	Students will be able to find unit vectors, and will be able to evaluate the dot product, also using the dot product to find new vectors.	5 days

Teacher Notes:

Additional Resources

Click links below to access additional resources used to design this unit:

Content Area: Pre-Calculus	
Unit Title: Unit #4- Linear Systems and Matrices	
Grade Level: 11th-12th	
<p>Unit Summary: In this unit, students will solve systems of equations by substitution, elimination, and graphing. They will learn how to recognize when a system has no solution, infinitely many solutions, or exactly one solution, and will apply the techniques used for solving systems to multivariable systems, including partial fraction decomposition. Students will represent systems of equations with matrices, and will perform elementary row operations. They will also add, subtract, multiply, and find the inverse of matrices. Students will be able to find the determinants of square matrices and will apply them to finding areas of triangles, testing for collinear points, and solving systems of equations.</p> <p>Interdisciplinary Connections: Sports, Architecture, Various topics in Science, Finance and Accounting</p> <p>21st Century Themes and Skills: Creativity and Innovation, Communication and Collaboration, Critical Thinking & Problem Solving, Information, Media, and Technology Skills, Life and Career Skills.</p> <p>CRP1. Act as a responsible and contributing citizen and employee. CRP2. Apply appropriate academic and technical skills. CRP4. Communicate clearly and effectively. CRP6. Demonstrate creativity and innovation. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP9. Model integrity, ethical leadership and effective management. CRP11. Use technology to enhance productivity.</p>	
Standards (Content and Technology):	
CPI#:	Statement:
NJSLS.A-REI.C.8(+)	(+) Represent a system of linear equations as a single matrix equation in a vector variable.
NJSLS.A-REI.9(+)	(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimensions 3x3 or greater).
NJSLS.N-VM.6(+)	(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
NJSLS.N-VM.7(+)	(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
NJSLS.N-VM.8(+)	(+) Add, subtract, and multiply matrices of appropriate dimensions.
NJSLS.N-VM.9(+)	(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
NJSLS.N-VM.10(+)	(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
NJSLS.N-VM.11(+)	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
NJSLS.N-VM.12(+)	(+) Work with 2x2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
Tech Standard: 8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
Tech Standard: 8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.
Tech Standard: 8.2.12.C.4	Explain and identify interdependent systems and their functions.

<p>Unit Essential Question(s):</p> <ul style="list-style-type: none"> ● How do you use substitution and graphing to solve systems of equations? ● How do you use elimination to solve systems of equations? ● How do you solve systems of equations in more than two variables? ● How do you use matrices to solve systems of equations? ● How do you perform operations on matrices? ● How do you find and use the inverse of a square matrix? ● How do you find the determinant of a square matrix? ● How do you use matrices to solve systems of equations, find areas of triangles, and write coded messages? 	<p>Unit Enduring Understandings:</p> <ul style="list-style-type: none"> ● Solve systems of equations by substitution, elimination, and graphing. ● Recognize when a system has no solution, infinitely many solutions, or exactly one solution. ● Apply these techniques to multivariable systems, including partial fraction decomposition. ● Represent systems of equations with matrices, and to perform elementary row operations. ● Add, subtract, multiply, and find the inverse of matrices. ● Use matrix inverses to solve systems of equations. ● Find the determinants of square matrices and apply them to finding the areas of triangles, testing for collinear points, and solving system of equations using Cramer’s Rule. 		
<p>Unit Learning Targets/Objectives: <i>Students will...</i></p> <ul style="list-style-type: none"> ● Be able to solve systems of equations by substitution, elimination, and graphing. ● Be able to recognize when a system has no solution, infinitely many solutions, or exactly one solution. ● Be able to apply the techniques used for solving systems to multivariable systems, including partial fraction decomposition. ● Be able to represent systems of equations with matrices, and will perform elementary row operations. ● Be able to add, subtract, multiply, and find the inverse of matrices. ● Be able to find the determinants of square matrices and will apply them to finding areas of triangles, testing for collinear points, and solving systems of equations. 			
<p>Formative Assessments:</p> <ul style="list-style-type: none"> ● Quizzes ● Homework ● On spot checking for understanding activities ● Entry/Exit tickets <p>Summative/Benchmark Assessment(s):</p> <ul style="list-style-type: none"> ● Tests ● Projects <p>Resources/Materials (copy hyperlinks for digital resources): <i>Larson Precalculus with Limits</i> Teacher Resources www.njctl.org/courses/math/pre-calculus/</p>			
<p>Modifications:</p> <table border="0" style="width: 100%;"> <tr> <td style="vertical-align: top; width: 50%;"> <ul style="list-style-type: none"> ● Special Education Students <ul style="list-style-type: none"> ○ Allow errors ○ Rephrase questions, directions, and explanations ○ Allow extended time to answer questions, and permit drawing, as an explanation ○ Accept participation at any level, even one word </td> <td style="vertical-align: top; width: 50%;"> <ul style="list-style-type: none"> ● At-Risk Students <ul style="list-style-type: none"> ○ Provide extended time to complete tasks ○ Consult with Guidance Counselors and follow I&RS procedures/action plans ○ Consult with classroom teacher(s) for specific behavior interventions ○ Provide rewards as necessary </td> </tr> </table>		<ul style="list-style-type: none"> ● Special Education Students <ul style="list-style-type: none"> ○ Allow errors ○ Rephrase questions, directions, and explanations ○ Allow extended time to answer questions, and permit drawing, as an explanation ○ Accept participation at any level, even one word 	<ul style="list-style-type: none"> ● At-Risk Students <ul style="list-style-type: none"> ○ Provide extended time to complete tasks ○ Consult with Guidance Counselors and follow I&RS procedures/action plans ○ Consult with classroom teacher(s) for specific behavior interventions ○ Provide rewards as necessary
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Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)
Solving Systems of Equations	Students will be able to use the methods of substitution and graphing to solve systems of equations in two variables.	2 days
Systems of Linear Equations in Two Variables	Students will be able to use the method of elimination to solve systems of linear equations and will graphically interpret the solutions.	3 days
Multivariable Linear Systems	Students will be able to use back-substitution and Gaussian elimination to solve linear systems in row-echelon form.	4 days
Matrices and Systems of Equations	Students will be able to write matrices, perform elementary row operations on matrices, and will use matrices to solve linear systems of equations.	4 days
Operations with Matrices	Students will be able to add and subtract matrices, multiply matrices by scalars, and multiply two matrices together.	5 days
The Inverse of a Square Matrix	Students will be able to verify that two matrices are inverses of each other and will use Gauss-Jordan elimination to find inverses of matrices.	3 days
The Determinant of a Square Matrix	Students will be able to find the determinants of 2x2 matrices, and use that knowledge to find the determinants of other square matrices.	2 days
Applications of Matrices and Determinants	Students will be able to use determinants to find areas of triangles, to determine whether points are collinear, and will use	2 days

	Cramer's Rule to solve linear equations.	
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Teacher Notes:

Additional Resources

Click links below to access additional resources used to design this unit:

Content Area: Pre-Calculus	
Unit Title: Unit #5- Sequences, Series, and Probability	
Grade Level: 11th-12th	
<p>Unit Summary: In this unit, students will study and analyze the patterns of sequences and series. They will represent sequences and series in different ways, find the nth term of different sequences, and will use summation notation. Students will compare and contrast arithmetic and geometric sequences, and will use the proven formulas to find the nth term of both types of sequences. They will be introduced to the Binomial Theorem in this unit, and will learn how to expand binomials of degree n with the assistance of Pascal's Triangle. Students will also be able to use counting principles and will determine the probabilities of multiple events in real-world applications.</p> <p>Interdisciplinary Connections: Sports, Architecture, Various topics in Science, Finance and Accounting</p> <p>21st Century Themes and Skills: Creativity and Innovation, Communication and Collaboration, Critical Thinking & Problem Solving, Information, Media, and Technology Skills, Life and Career Skills.</p> <p>CRP1. Act as a responsible and contributing citizen and employee. CRP2. Apply appropriate academic and technical skills. CRP4. Communicate clearly and effectively. CRP6. Demonstrate creativity and innovation. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP9. Model integrity, ethical leadership and effective management. CRP11. Use technology to enhance productivity.</p>	
Standards (Content and Technology):	
CPI#:	Statement:
NJSLS.A-SEE.B.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
NJSLS.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
NJSLS.S-CP.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
NJSLS.S-CP.7	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
NJSLS.S-CP.8(+)	(+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)$, and interpret the answer in terms of the model.
NJSLS.S-CP.9(+)	(+) Use permutations and combinations to compute probabilities of compound events and solve problems.
Tech Standard: 8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
Tech Standard: 8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.
Tech Standard: 8.2.12.C.4	Explain and identify interdependent systems and their functions.
Unit Essential Question(s): <ul style="list-style-type: none"> How do you represent a sequence of numbers or the sum of a sequence? How do you find the nth term or partial sum of an arithmetic sequence? How do you find terms and sums of geometric sequences? 	Unit Enduring Understandings: <ul style="list-style-type: none"> Represent sequences and series, including summation notation. Model and find sums of arithmetic and geometric sequences.

<ul style="list-style-type: none"> How do you count the number of ways in which an event can occur, and how do you find the probability that a series of events will occur? 	<ul style="list-style-type: none"> Count possible outcomes and determine the probabilities of multiple events.
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Unit Learning Targets/Objectives:
Students will...

- Be able to represent sequences and series as equations, including summation notation.
- Be able to model and find sums of arithmetic and geometric sequences.
- Be able to count possible outcomes and determine the probabilities of multiple events.

Formative Assessments:

- Quizzes
- Homework
- On spot checking for understanding activities
- Entry/Exit tickets

Summative/Benchmark Assessment(s):

- Tests
- Projects

Resources/Materials (copy hyperlinks for digital resources):
Larson Precalculus with Limits Teacher Resources
www.njctl.org/courses/math/pre-calculus/

Modifications:

<ul style="list-style-type: none"> Special Education Students <ul style="list-style-type: none"> Allow errors Rephrase questions, directions, and explanations Allow extended time to answer questions, and permit drawing, as an explanation Accept participation at any level, even one word Consult with Case Managers and follow IEP accommodations/modifications English Language Learners <ul style="list-style-type: none"> Assign a buddy, same language or English speaking Allow errors in speaking Rephrase questions, directions, and explanations Allow extended time to answer questions Accept participation at any level, even one word 	<ul style="list-style-type: none"> At-Risk Students <ul style="list-style-type: none"> Provide extended time to complete tasks Consult with Guidance Counselors and follow I&RS procedures/action plans Consult with classroom teacher(s) for specific behavior interventions Provide rewards as necessary Gifted and Talented Students <ul style="list-style-type: none"> Provide extension activities Build on students' intrinsic motivations Consult with parents to accommodate students' interests in completing tasks at their level of engagement
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Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)
Sequences and Series	Students will be able to represent sequences and series, including summation notation.	2 days

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Arithmetic Sequences and Partial Sums	Students will be able to model and find sums of arithmetic sequences, find the n th term, and will use the appropriate formula to find partial sums.	4 days
Geometric Sequences and Series	Students will be able to model and find sums of geometric sequences, find the n th term, and will use the appropriate formula to find partial sums.	4 days
The Binomial Theorem	Students will be able to expand binomials by using the Binomial Theorem and Pascal's Triangle.	3 days
Counting Principles	Students will be able to count possible outcomes of different real-world situations.	4 days
Probability	Students will be able to determine the probabilities of multiple events in different real-world situations.	3 days
<p>Teacher Notes:</p> <p>Additional Resources Click links below to access additional resources used to design this unit:</p>		

Content Area: Pre-Calculus	
Unit Title: Unit #6- Conic Sections	
Grade Level: 11th-12th	
<p>Unit Summary: In this unit, students will write equations of circles and parabolas, graph them on the coordinate plane, and use the distance formula to solve problems. They will write equations of ellipses, use properties of ellipses to model problems, and find eccentricities of ellipses. Students will be presented with problems involving conic sections, and will eventually classify a conic by its equation in general form. They will learn how to rotate a conic section in order to simplify its equation and write and graph equations in parametric and polar forms. Students will use polar coordinates to represent and solve problems involving conic sections.</p> <p>Interdisciplinary Connections: Sports, Architecture, Various topics in Science, Finance and Accounting</p> <p>21st Century Themes and Skills: Creativity and Innovation, Communication and Collaboration, Critical Thinking & Problem Solving, Information, Media, and Technology Skills, Life and Career Skills.</p> <p>CRP1. Act as a responsible and contributing citizen and employee. CRP2. Apply appropriate academic and technical skills. CRP4. Communicate clearly and effectively. CRP6. Demonstrate creativity and innovation. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP9. Model integrity, ethical leadership and effective management. CRP11. Use technology to enhance productivity.</p>	
Standards (Content and Technology):	
CPI#:	Statement:
NJSLS.G-GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
NJSLS.G-GPE.A.2	Derive the equation of a parabola given a focus and directrix.
NJSLS.G-GPE.A.3(+)	(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
Tech Standard: 8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
Tech Standard: 8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.
Tech Standard: 8.2.12.C.4	Explain and identify interdependent systems and their functions.
<p>Unit Essential Question(s):</p> <ul style="list-style-type: none"> ● How do you recognize each conic section and solve problems involving parabolas? ● How do you solve problems involving ellipses? ● How do you solve problems involving hyperbolas, classify a conic from its general equation, and eliminate the xy term from the general equation of a conic section? ● How do you write equations to describe the motion of a point in a plane? ● How do you describe the position of a point in a plane using distance and angles rather than x and y coordinates? 	<p>Unit Enduring Understandings:</p> <ul style="list-style-type: none"> ● The definition of a circle and the distance formula. ● Solve problems involving conic sections, eventually classifying a conic by its equation in general form. ● Rotate a conic section in order to simplify its equation. ● Write and graph equations in parametric and polar forms. ● Use polar coordinates to represent and solve problems involving conic sections.

- How do you sketch graphs of polar equations?
- How do you represent conic sections in polar coordinates?

Unit Learning Targets/Objectives:

Students will...

- Be able to use the definition of a circle and use the distance formula to find equations of circles.
- Be able to solve problems involving conic sections, eventually classifying a conic by its equation in general form.
- Be able to rotate a conic section in order to simplify its equation.
- Be able to write and graph equations in parametric and polar forms.
- Be able to use polar coordinates to represent and solve problems involving conic sections.

Formative Assessments:

- Quizzes
- Homework
- On spot checking for understanding activities
- Entry/Exit tickets

Summative/Benchmark Assessment(s):

- Tests
- Projects

Resources/Materials (copy hyperlinks for digital resources):

Larson Precalculus with Limits Teacher Resources
www.njctl.org/courses/math/pre-calculus/

Modifications:

- | | |
|---|--|
| <ul style="list-style-type: none"> ● Special Education Students <ul style="list-style-type: none"> ○ Allow errors ○ Rephrase questions, directions, and explanations ○ Allow extended time to answer questions, and permit drawing, as an explanation ○ Accept participation at any level, even one word ○ Consult with Case Managers and follow IEP accommodations/modifications ● English Language Learners <ul style="list-style-type: none"> ○ Assign a buddy, same language or English speaking ○ Allow errors in speaking ○ Rephrase questions, directions, and explanations ○ Allow extended time to answer questions ○ Accept participation at any level, even one word | <ul style="list-style-type: none"> ● At-Risk Students <ul style="list-style-type: none"> ○ Provide extended time to complete tasks ○ Consult with Guidance Counselors and follow I&RS procedures/action plans ○ Consult with classroom teacher(s) for specific behavior interventions ○ Provide rewards as necessary ● Gifted and Talented Students <ul style="list-style-type: none"> ○ Provide extension activities ○ Build on students' intrinsic motivations ○ Consult with parents to accommodate students' interests in completing tasks at their level of engagement |
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Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)
Circles and Parabolas	Students will be able to write equations of circles and parabolas, graph them on the coordinate plane,	4 days

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	and use the distance formula to solve problems.	
Ellipses	Students will be able to write equations of ellipses, use properties of ellipses to model problems, and find eccentricities of ellipses.	4 days
Hyperbolas and Rotation of Conics	Students will be able to write equations of hyperbolas, find asymptotes, and perform rotations of conics.	5 days
Parametric Equations	Students will be able to evaluate sets of parametric equations and graph the curves represented by the sets.	4 days
Polar Coordinates	Students will be able to plot points on the polar coordinate system and convert points from rectangular to polar.	4 days
Graphs of Polar Equations	Students will be able to graph polar coordinates using symmetry and zeros as aids.	3 days
Polar Equations of Conics	Students will be able to write and graph equations of conics in polar form.	5 days
<p>Teacher Notes:</p> <p>Additional Resources Click links below to access additional resources used to design this unit:</p>		

Content Area: Pre-Calculus	
Unit Title: Unit #7- Introduction to Limits	
Grade Level: 11th-12th	
<p>Unit Summary: In this unit, students will use techniques for calculating the limit of a graph at a given value of x. They will find the slope of a graph at a given point and calculate the derivative of a function. Students will also be able to find the limit of functions at infinity and find the limit of sequences. They will apply the techniques of finding limits to find the area of a region bounded by a function.</p> <p>Interdisciplinary Connections: Sports, Architecture, Various topics in Science, Finance and Accounting</p> <p>21st Century Themes and Skills: Creativity and Innovation, Communication and Collaboration, Critical Thinking & Problem Solving, Information, Media, and Technology Skills, Life and Career Skills.</p> <p>CRP1. Act as a responsible and contributing citizen and employee. CRP2. Apply appropriate academic and technical skills. CRP4. Communicate clearly and effectively. CRP6. Demonstrate creativity and innovation. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP9. Model integrity, ethical leadership and effective management. CRP11. Use technology to enhance productivity.</p>	
Standards (Content and Technology):	
CPI#:	Statement:
NJSLS.S-ID.C.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
NJSLS.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
Tech Standard: 8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
Tech Standard: 8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.
Tech Standard: 8.2.12.C.4	Explain and identify interdependent systems and their functions.
<p>Unit Essential Question(s):</p> <ul style="list-style-type: none"> How do you find and interpret the limit of a function for a certain value of x? How do you evaluate limits that cannot be solved through use of direct substitution? How do you find the slope of a graph at any single point? How do you find the limits of functions at infinity and the limits of sequences? How do you approximate and find exact areas of plane regions defined by functions? 	<p>Unit Enduring Understandings:</p> <ul style="list-style-type: none"> Use techniques for calculating the limit of a graph at a given value of x. Find the slope of a graph at a given point and calculate the derivative of a function. Find the limit of functions at infinity and the limits of sequences. Apply these techniques to find the area of a region bounded by a function.
<p>Unit Learning Targets/Objectives:</p> <p><i>Students will...</i></p> <ul style="list-style-type: none"> Be able to use techniques for calculating the limit of a graph at a given value of x. Be able to find the slope of a graph at a given point and calculate the derivative of a function. Be able to find the limit of functions at infinity and find the limit of sequences. 	

- Be able to apply the techniques of finding limits to find the area of a region bounded by a function.

Formative Assessments:

- Quizzes
- Homework
- On spot checking for understanding activities
- Entry/Exit tickets

Summative/Benchmark Assessment(s):

- Tests
- Projects

Resources/Materials (copy hyperlinks for digital resources):

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www.njctl.org/courses/math/pre-calculus/

Modifications:

- Special Education Students
 - Allow errors
 - Rephrase questions, directions, and explanations
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- At-Risk Students
 - Provide extended time to complete tasks
 - Consult with Guidance Counselors and follow I&RS procedures/action plans
 - Consult with classroom teacher(s) for specific behavior interventions
 - Provide rewards as necessary
- Gifted and Talented Students
 - Provide extension activities
 - Build on students' intrinsic motivations
 - Consult with parents to accommodate students' interests in completing tasks at their level of engagement

Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)
Introduction to Limits	Students will be able to use the definition of limits to estimate limits and determine whether limits of functions exist or to evaluate limits.	4 days
Techniques for Evaluating Limits	Students will be able to evaluate limits of functions by using the dividing and the rationalizing techniques, and will evaluate one-sided limits.	4 days
The Tangent Line	Students will be able to understand and use the tangent line to approximate the slope of a graph at a certain point, also using derivatives to find slopes of graphs.	3 days

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Limits at Infinity and Limits of Sequences	Students will be able to evaluate limits of functions at infinity and will find limits of sequences.	4 days
Area of A Region	Students will be able to find limits of summations and will use rectangles to approximate areas of plane regions.	4 days
Teacher Notes:		
Additional Resources Click links below to access additional resources used to design this unit:		